



Figure 12: The bias (a) and the RMS error (b) of the computed fundamental matrix for the standard deviation σ (pixels) of the noise added to the data in Fig. 11 over 10000 independent trials. 1) LS, 2) iterative reweight, 3) Taubin, 4) renormalization, 5) HyperLS, 6) hyper-renormalization, 7) ML, 8) hyperaccurate correction of ML. The dotted line in (b) indicates the KCR lower bound.

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